Monte Carlo study of DNA condensation phenomena

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Outline

- Overview
- Poisson-Boltzmann equation
- Metropolis Monte Carlo
- Theoretical models: DNA condensation
- Experimental and theoretical results
- Conclusions and future directions

Origin of Poisson-Boltzmann equation

Poisson equation

$$\nabla^2 \Phi_k(\mathbf{r}) = \frac{-4\pi\rho_k(\mathbf{r})}{\varepsilon_k}$$

Gauss' law

$$\Phi_E = k_e q \oint \frac{dA\cos(\theta)}{r^2} = k_e q \oint d\Omega = 4\pi k_e q = \frac{q}{\varepsilon_0}$$

combine…

$$\nabla^2 \Phi_1(\mathbf{r}) = \sum_i \frac{q_i}{\varepsilon_1 \left|\mathbf{r} - \mathbf{r}_i\right|} = \sum_i \frac{-4\pi q_i}{\varepsilon_1} \delta(\mathbf{r} - \mathbf{r}_i)$$

Boltzmann distribution law

$$e^{-W_i(\mathbf{r})/(k_B T)}$$

For positively and negatively charged ions the work functions are:

$$W_1(\mathbf{r}) = +e_c\Phi(\mathbf{r}) \qquad W_2(\mathbf{r}) = -e_c\Phi(\mathbf{r})$$

$$M_+ = Me^{-e_c\Phi(\mathbf{r})/(k_B T)} \qquad M_- = Me^{+e_c\Phi(\mathbf{r})/(k_B T)}$$

Let ionic strengths, i.e. number of positive and negative ions, be equal $M^+=M^-$. $+e_c$ is a charge.

$$\rho(\mathbf{r}) = Me_c e^{-e_c\Phi(\mathbf{r})/(k_B T)} - Me_c e^{+e_c\Phi(\mathbf{r})/(k_B T)} = -2Me_c \sinh\left(\frac{e_c\Phi(\mathbf{r})}{k_B T}\right)$$

Poisson-Boltzmann equation

$$\nabla^2\Phi(\mathbf{r}) = \frac{-4\pi\rho(\mathbf{r})}{\varepsilon} = \left(\frac{8\pi M e_c}{\varepsilon}\right)\sinh\left(\frac{e_c\Phi(\mathbf{r})}{k_B T}\right)$$

$$-\nabla\cdot(\varepsilon(\mathbf{r})\nabla\Phi(\mathbf{r})) + \kappa^2(\mathbf{r})\left(\frac{k_B T}{e_c}\right)\sinh\left(\frac{e_c\Phi(\mathbf{r})}{k_B T}\right) = 4\pi\sum_i q_i\delta(\mathbf{r}-\mathbf{r}_i)$$

Metropolis scheme

Trial move

Calculate energy

deltaE = Enew - Eold

if Enew < Eold

    move

else Enew > Eold

    if random variable[0:1] < exp(-deltaE/kT)

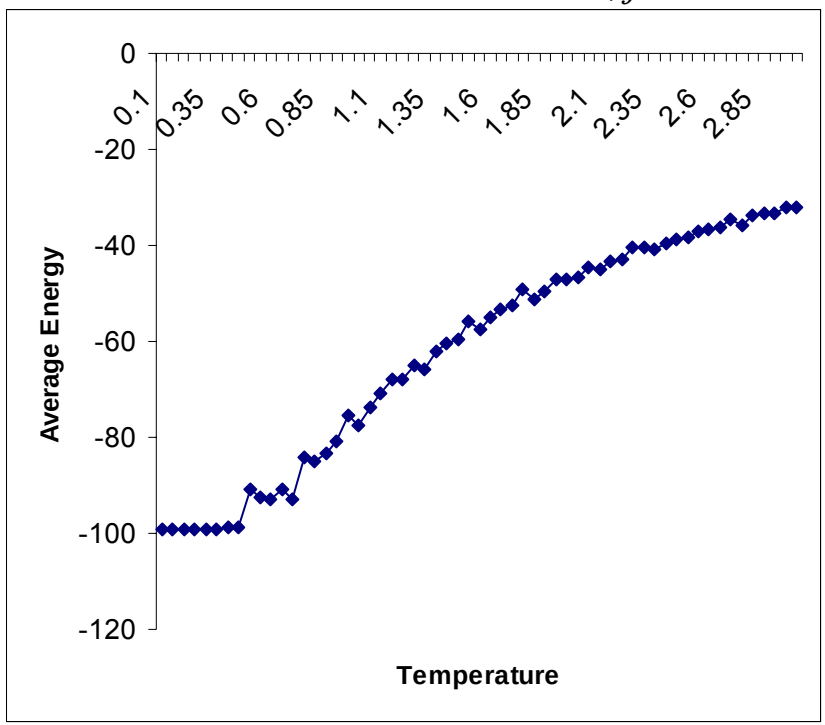
        move

    else

        reject move

Monte Carlo example using Metropolis condition for 1D Ising system: *average energy increases as temperature increases*

$$E_\nu = -\mu H \sum_i s_i - \frac{1}{2} \sum_{i,j} J_{ij} s_i s_j$$



Theoretical models

- Mean field approach (Poisson-Boltzmann and Debye-Hückel)
- Explicit ions (Lyubartsev)
- Hydration (Rau and Parsegian)
- Cylindrical models:
  - line charge, helical, cylinder with major and minor grooves, helicity involved in observed attraction

Error in PB greater than 1% in close-packing region

Table 3-1. Potential [$k_B T / e_0$] at the Surface of a Sphere of the Radius of 1 Å Calculated by Solving Equation 3-11 for the Bulk Model and Cell Models with Different Concentration of 1-1 Electrolyte[a]

C_s, M	κ, Å^{-1}	R, Å								
		10	20	40	70	100	120	150	300	infinite
0.001	0.0104	-5.32 (32)	-6.26 (13)	-6.78 (4)	-6.95 (2)	-7.00 (1)	-7.01 (1)	-7.03 (1)	-7.05 (0)	-7.04
0.010	0.033	-5.32 28)	-6.25 (8)	-6.67 (1)	-6.77 (0)	-6.78 (0)	-6.79 (0)	-6.79 (0)	-6.80 (0)	-6.80
0.025	0.052	-5.32(22)	-6.20(5)	-6.50 (0)	-6.55 (0)	-6.56 (-1)	-6.56 (-1)	-6.56 (-1)	-6.56	
0.050	0.073	-5.31 (18)	-6.09 (3)	-6.29 (0)	-6.30 (-1)	-6.31 (-1)	-6.31 (-1)	-6.31 (-1)	-6.31	
0.100	0.104	-5.29 (12)	-5.89 (1)	-5.99 (-1)	-6.00 (-1)	-6.00 (-1)	-6.00 (-1)	-6.00 (-1)	-6.00	
0.200	0.147	-5.20 (7)	-5.59 (0)	-5.62 (0)	-5.63(-1)	-5.63(-1)	-5.63(-1)	-5.63(-1)	-5.6	
0.500	0.232	-4.91 (3)	-5.06 (0)	-5.07 (-1)	-5.07 (-1)	-5.07 (-1)	-5.07 (-1)	-5.07 (-1)	-5.07	

[a]The relative difference in potentials (in percent) obtained by the infinite and cell models is given i
The shaded area represents the region of parameters (salt concentration and cell radius), for which t
solutions of the PB equation is more than 1 %.

(Petrov, 2004)

PM := Primitive model of DNA in cylindrical shell

(Lyubartsev *et al.*, 1995)

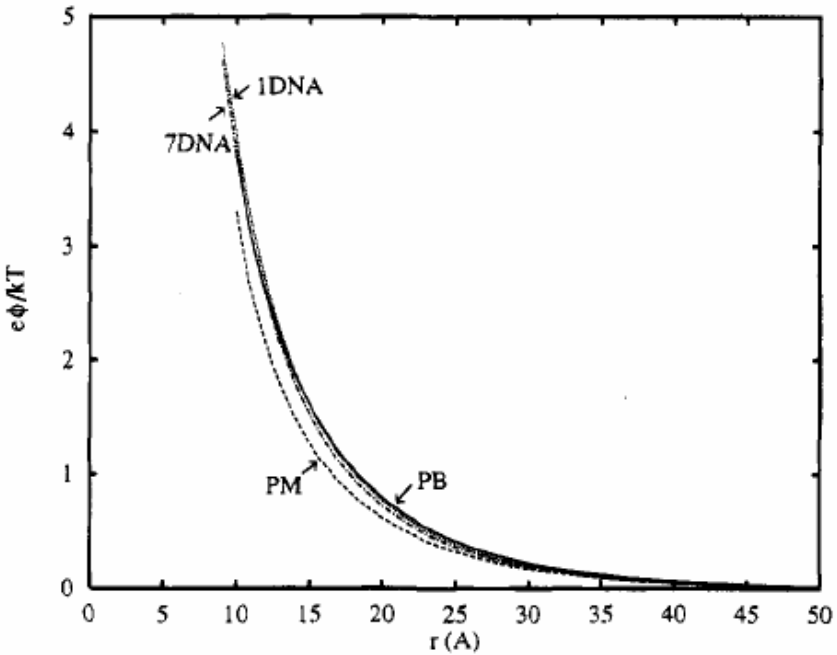
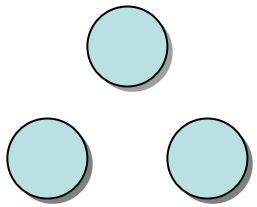


Figure 3. Electrostatic potential around DNA in 0.1 M 1:1 electrolyte solution. Notation as in Figure 2.

Trigonal and hexagonal cylindrical packing



Trigonal packing

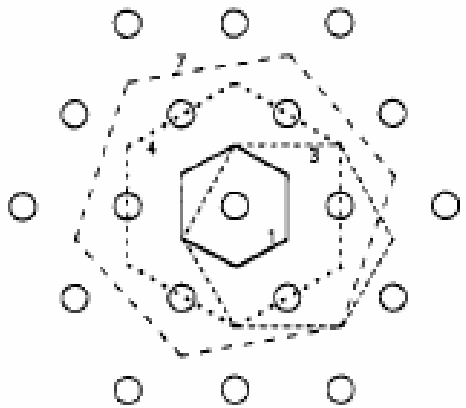


Figure 1. Hexagonally ordered DNA system with Monte Carlo cells containing different numbers of DNA.

(Lyubartsev *et al.,* 1995)

Theoretical and experimental results

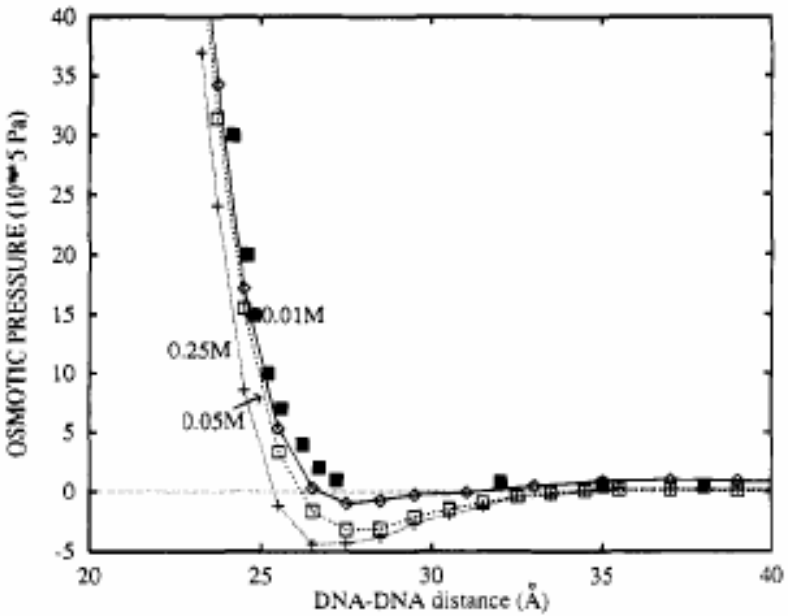


Figure 10. Osmotic pressure in the ordered DNA system in equilibrium with a 2:1 electrolyte bulk phase at different salt concentrations. Diameter of all ions is 4 Å. Filled squares are experimental results for 0.05 M $MnCl_2$.[5]

(Lyubartsev *et al.,* 1995)

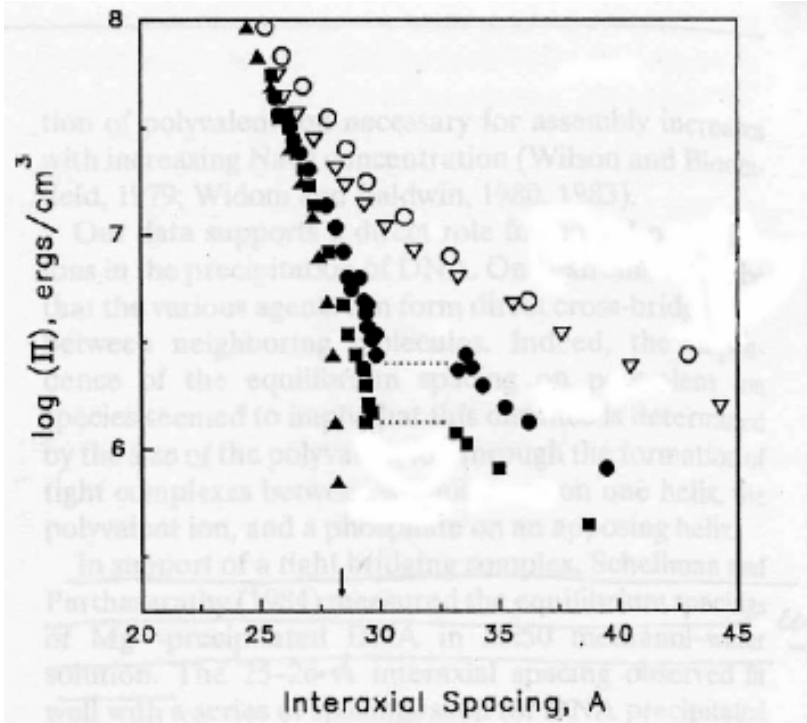
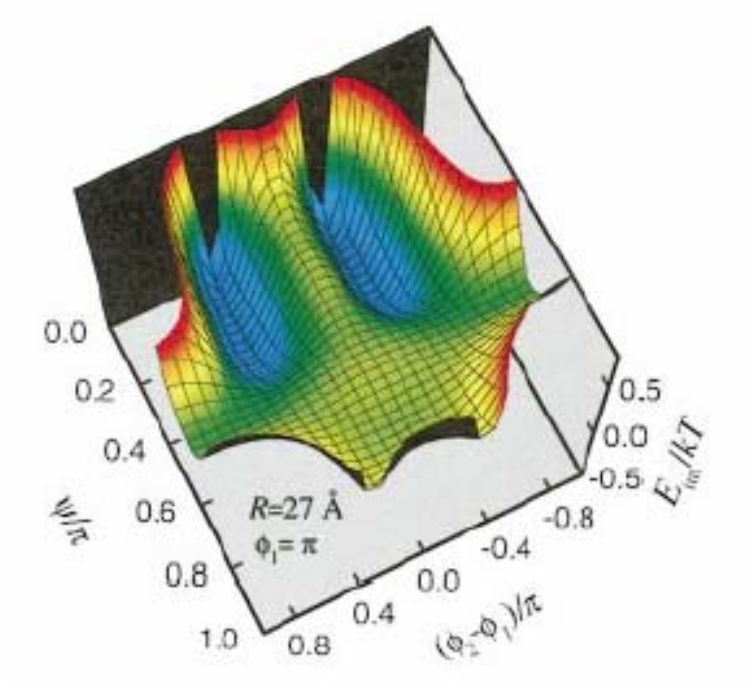


FIGURE 6 Intermolecular force curves for a series of cobalt hexammine concentrations in 0.25 M NaCl, 10 mM TrisCl (pH 7.5), at 20°. Spontaneous precipitation occurs at 17 mM Co^{3+} at this ionic strength. 0 mM $Co(NH_3)_6Cl_3$ (○). 2 mM (▽). 8 mM (●). 12 mM (■). 20 mM (▲). Abrupt transitions from one kind of force curve to another indistinguishable from the 20 mM data are clearly apparent for the 8 and 12 mM data.

(Rau & Parsegian, 1992)

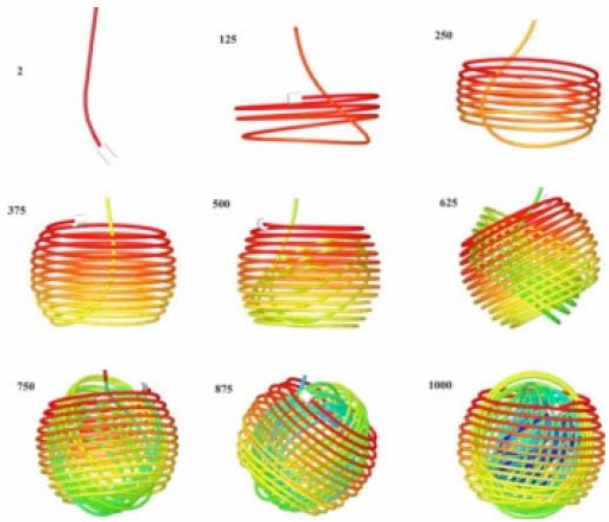
Theoretical and experimental results (p2)

Energy landscapes for interaction between two crossed *B*-DNA-like helices. Plotted interaxial angle ψ and difference in about-axis rotation angles φ_2-φ_1. (Kornyshev *et al.*, 2000)



Conclusions and future directions

- Force field improvement for DNA condensation in presence of ions
- Polymer physics of viral capsid DNA folding
- Future modeling



(LaMarque *et al*., 2003)